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Elastic Response of Accelerating Launch Vehicles Subjected to Varying Control Pulses

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Introduction

MODERN launch vehicles employ large forces for better control performance that result in significant elastic excitation of the lightly damped launch vehicle. These responses cause excitation to payload, corrupt the feedback sensor signal, and produce large dynamic bending moments on the structure. Recently the author considered the effect of trajectory accelerations and found significant changes in the free-vibration behavior of launch vehicles.[1,2] In the present study, the author analyzes the effects of control pulse duration and trajectory accelerations on the overall elastic response of typical launch vehicle geometry.[3] Accelerations at the tip and the maximum dynamic bending moment are obtained for different values of pulse duration in such a way that total momentum of the pulse is constant. The vehicle is modeled as a stepped beam of constant property segments. Transverse vibration of such beams can be adequately modeled using the Euler–Bernoulli beam theory.[2]

Formulation and Solution

The elastic response of a lightly damped structure can be adequately obtained using the mode superposition approach[4] as follows:

$$w(x,t) = \sum_{r}^{\infty} \frac{1}{\omega_r} \int_0^t F_r(\tau)\exp[-\xi_r\omega_r(t-\tau)]\sin\omega_r(t-\tau)\,\mathrm{d}\tau \tag{1}$$

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where ω_r is undamped natural frequency, ξ_r is equivalent viscous damping, and $F_r(\tau)$ is excitation force in rth mode. The modal solution is obtained from the free-vibration equations given by

$$\left(\frac{\partial^4 w_i}{\partial \bar{x}_i^4}\right) - a_i\left(\frac{\partial^2 w_i}{\partial \bar{x}_i^2}\right) + \gamma_i^4 w_i = 0 \tag{2}$$

Here, γ_i $[=(\rho A)_i\omega^2 L_o^4/(EI)_i]^{(1/4)}$ is the dimensionless frequency parameter for the ith segment. Variable $\bar{x}_i$ $(=x_i/L_o)$ takes values from 0 to $\bar{L}_i$ $(=L_i/L_o)$ for all segments, where L_i is the length of each segment and a_i $(=P_i L_o^2/EI_o)$ is the nondimensional axial inertia force parameter due to trajectory acceleration. A new frequency parameter λ for the complete vehicle is defined as

$$\lambda^4 = \left[(\rho A)_o\omega^2 L_o^4/(EI)_o\right] \tag{3}$$

where ρA_o is the average mass per length and EI_o is the maximum bending rigidity. The general solution[1] of Eq. (2) can be written as

$$w_i = A_i\cosh\lambda_1\bar{x}_i + B_i\sinh\lambda_1\bar{x}_i + C_i\cos\lambda_2\bar{x}_i + D_i\sin\lambda_2\bar{x}_i \tag{4}$$

where A_i, B_i, C_i, and D_i, are arbitrary constants and λ_1 and λ_2 are roots of characteristic equation for each segment and are obtained as

$$\lambda_1^2 = \frac{\left(a_i^2 + 4\gamma_i^4\right)^{\frac{1}{2}} - a_i}{2} \tag{5}$$

$$\lambda_2^2 = \frac{\left(a_i^2 + 4\gamma_i^4\right)^{\frac{1}{2}} + a_i}{2} \tag{6}$$

Enforcing boundary conditions at two ends and continuity conditions between N segments on displacement, slope, bending moment, and shear force results in a total of $4N$ conditions for $4N$ unknowns and a $4N \times 4N$ characteristic determinant whose zeroes give the natural frequency λ. The corresponding normalized modes are obtained by substituting eigenvalue in the characteristic equations.[1]

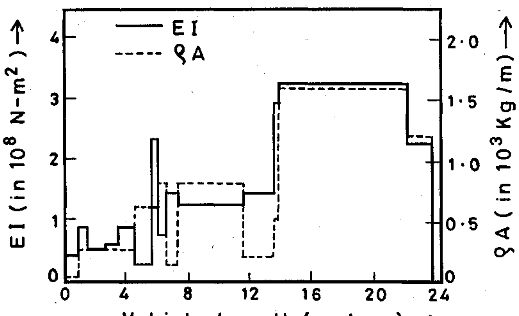

Fig. 1 Structural configuration of a typical launch vehicle.

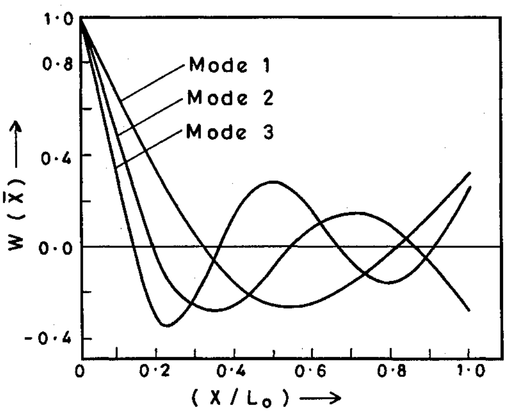

Fig. 2 First three vibration mode shapes of the launch vehicle.

Table 1 Free-vibration characteristics of a typical launch vehicle

	$n_o g = 0\,g$			$n_o g = 10\,g$			$n_o g = 20\,g$		
Parameter	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
Frequency, Hz	3.536	8.815	16.87	3.484	8.749	16.78	3.429	8.682	16.69
Modal mass, kg	997.6	665.1	598.6	997.6	665.1	598.6	997.6	665.1	598.6

Table 2 Elastic response for different acceleration and pulse duration

	Net impulse applied $= 10^4$ N-s					
	$n_o g = 0\,g$		$n_o g = 10\,g$		$n_o g = 20\,g$	
Time, s	a_e	M_e	a_e	M_e	a_e	M_e
10.00	0.062	2.63×10^6	0.041	2.46×10^6	0.029	2.24×10^6
5.00	0.123	5.25×10^6	0.081	4.82×10^6	0.059	4.48×10^6
2.00	0.309	1.31×10^7	0.203	1.30×10^7	0.147	1.12×10^7
1.00	0.528	2.62×10^7	0.950	1.96×10^7	0.493	1.72×10^7
0.50	2.330	3.14×10^7	1.070	2.93×10^7	0.937	3.32×10^7
0.10	5.690	1.37×10^8	3.680	1.23×10^8	2.150	1.20×10^8
0.01	17.30	2.07×10^8	16.60	1.84×10^8	6.020	1.45×10^8
0.00	21.20	2.21×10^8	19.10	1.85×10^8	6.070	1.54×10^8

Sample Example, Numerical Results, and Discussion

A typical launch vehicle[3] (Fig. 1) is considered for analysis and is modeled using 14 spanwise segments ($\omega_1 = 22.217$ against 22.373 rad). The results for free and forced vibration are obtained for three values of trajectory acceleration $n_o g$ (0, 10, and 20 g) and a representative structural damping factor ξ_r of 0.05. In practice, structural modes higher than three do not contribute much to the overall dynamic response [Eq. (1)], and thus only the first three modes of vibration are included in the analysis. Results in terms of tip acceleration and peak dynamic bending moment are obtained for a rectangular input pulse having different durations from 10 to 0.0 s, with the same momentum of 10^4 N-s imparted to the vehicle at the end of each pulse. Table 1 and Fig. 2 present free-vibration results for different values of trajectory acceleration, and it can be seen that there is a slight reduction in the cyclic frequency (~3%). Modal mass remains constant with change in $n_o g$, due mainly to constant stiffness and mass distributions. The results in Table 2 show that as the pulse duration decreases from 10 to 0 s, the elastic response in terms of tip acceleration (a_e in g units) and peak bending moment (M_e in N-M units) approaches the impulse response values and that the tip acceleration reduces significantly with increase in trajectory acceleration. Therefore it seems that a smaller magnitude pulse acting for a longer duration is desirable for lower elastic response. Furthermore, one can interpret the duration of the pulse as a good indicator of the setting time of the short-period dynamics of the rigid vehicle, and it can be seen that a shorter pulse duration with higher magnitude is desirable for better short-period performance. Therefore the author believes that an optimum value for the control force magnitude and its duration can be arrived at for a given launch vehicle by constructing a suitable objective cost function, including both the elastic and control requirements.

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